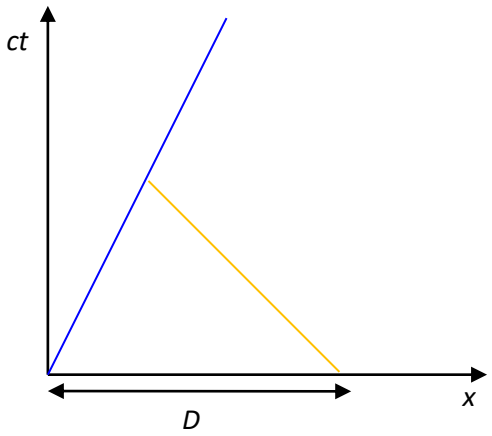
Teacher notes
Topic A

An instructive relativity problem.

The blue line is the worldline of a rocket moving past earth with speed *v*. A laser beam is emitted from *x* = *D* at *t* = 0. When does the beam get to the rocket according to rocket clocks?



Consider the events:

E_1 = beam is emitted

E_2 = beam arrives at rocket

For earth, the time between these events is Δt and $\Delta x = v\Delta t$ since the rocket has moved closer to the launch point in the time Δt. Hence

$$\Delta t' = \gamma(\Delta t - \frac{v}{c^2}\Delta x)$$
$$= \gamma(\Delta t - \frac{v}{c^2}v\Delta t) = \gamma\Delta t(1 - \frac{v^2}{c^2})$$
$$= \gamma\Delta t\frac{1}{\gamma^2} = \frac{\Delta t}{\gamma}$$

According to earth $\Delta t = \frac{D}{v+c}$. This is because the distance between the launch point and the rocket is decreasing at a rate $v+c$. This does not violate the speed of light being the maximum possible. No material body is moving at this speed. Hence $\Delta t = \frac{1}{\gamma}\frac{D}{v+c}$.